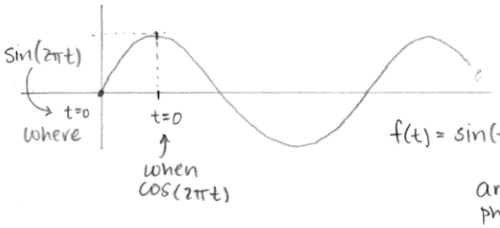
Physical Sciences 3
Lectures 9 and 10 – March 5, 2008 – Fourier Series and RC Filters

FOURIER SERIES

Theorem: any periodic function and therefore any function over a finite range can be expressed as a series of sines and cosines.

$f(x) = f(x + 2\pi)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$$



$f(t) = \sin(t + \phi)$ ← arbitrary phase shift

$\phi = \frac{\pi}{2}$ makes $\cos(t)$

$\sin(t+\phi) = \sin t \cos\phi + \cos t \sin\phi$

NOW in terms of angular frequency, $\omega = 2\pi f$

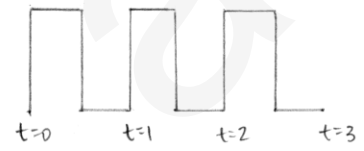
$\sin(2\pi f t)$ or $\cos(2\pi f t)$ in radians
$\sin(\omega t)$ or $\cos(\omega t)$

EXAMPLE FUNCTIONS

- Sawtooth function

$$f(t) = \sin(2\pi t) + \frac{\sin(2\times 2\pi t)}{2} + \frac{\sin(3\times 2\pi t)}{3} + \cdots$$

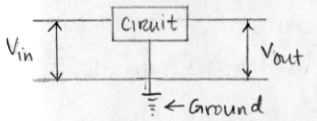
- squarewave function



$$f(t) = \sin(2\pi t) + \frac{\sin(3\times 2\pi t)}{3} + \frac{\sin(5\times 2\pi t)}{5}$$

THE CONNECTION TO:

It is useful to make electrical circuits that respond differently to different frequencies. Electrical circuits transform an input wave form into an output wave form



* this ΔV or $V_{in} - V_{out}$ is time-varying voltage (~) gives AC – alternating current – circuit.

* When a circuit has a resistor and capacitor to ground, called an RC filter

RC FILTERS

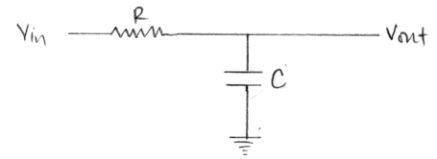
possible: amplitude Δ, phase Δ

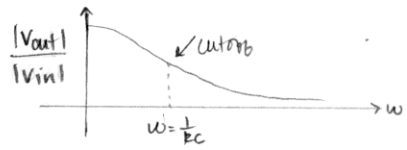
* Isolation of frequencies *

Two types of filters: HIGH & LOW PASS

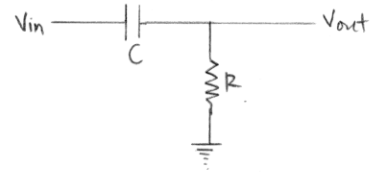
the output amplitude is roughly half that of the input is the frequency $1/2\pi RC$
→ where $\omega = 1/RC$ cut-off

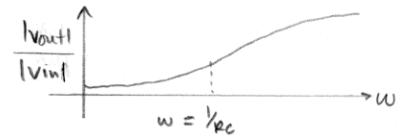
LOW PASS: frequencies below the cutoff frequency pass, those higher are attenuated





HIGH PASS: frequencies above the cutoff frequency pass, those lower are attenuated.





$$V_{in}(t) = V_0 \sin\omega t = IR + \frac{Q}{C}$$

$$V_{out}(t) = \frac{Q}{C} = A\sin\omega t + B\cos\omega t$$

$$V_{in}(t) = RC + \frac{d V_{out}(t)}{dt} + V_{out}(t)$$

$$\therefore V_0 \sin\omega t = (A - RCB\omega)\sin\omega t + (B + RCA\omega)\cos\omega t$$